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A Moving Single Station Passive Ranging Method by Interchangeable Relationship Between Frequency Shift and Path Difference

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Abstract

Based on the mathematical definition of Doppler shift equation, the commutative relation between Doppler shift and path difference can be obtained by using the difference of distance. By using the basic trigonometric function, a passive ranging solution based only on two consecutive Doppler frequency shift measurement can be obtained. Compared with the existing ranging solution obtained by frequency shift difference based on mathematical definition of Doppler rate of change, the new ranging solution still keeps the simple mathematical expression and has better calculation accuracy. However, by comparison, it can be found that: under the same measurement method, although the mathematical expression of the ranging solution is different, the ranging error still remains the same magnitude. At the same time, it is found that the moving single-station Doppler frequency shift ranging has three characteristics: frequency shift exchange, mean effect, and result lag.

Keywords: ***Single station location; Airborne passive positioning; Doppler shift；Doppler changing rate; Path difference equation; Ranging.***

1. Introduction

The existing research shows that the explicit solution of airborne passive ranging can be obtained by using the Doppler rate of change directly, and the location can be realized only by one measurement without the need to know the relative velocity between the target and the reconnaissance station. But it usually needs to be integrated with other measurement methods to achieve the positioning task, and the actual measurement of Doppler change rate is relatively difficult [1]-[3].

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One of the results of the author's early research on the passive ranging method based on Doppler rate of change is that, based on the mathematical definition, the Doppler rate of change is transformed into the ratio of the Doppler frequency difference to the time difference by using the difference method, and at the same time, by using the velocity vector relationship, an airborne Doppler passive ranging method based on two consecutive frequency shift measurement is derived [4], [5]. The result of this study is that there are both advantages and disadvantages. The advantages are that direct measurement of Doppler rate of change is not required, and there is no need to use other measurement methods. The disadvantages are that two measurements of the Doppler shift are required.

A recent research result of the author [6] is that an airborne passive ranging solution based on angle and frequency difference measurement is presented by using the reciprocal relation between frequency shift and path difference, and also by using the double base path difference locating method and the single base midpoint direction finding solution derived from this method [5]. The ranging method based on the measurement of angle and frequency difference requires two measurement methods at the same time, so it seems not superior. But research ideas tend to evolve gradually. In fact, the Doppler frequency shift function itself contains angle information, and some angle values can be obtained directly by using the Doppler frequency shift equation. The further study in this paper shows that if the basic trigonometric function relation is directly used on the basis of the commutation relation between frequency shift and path difference, the calculation formula of airborne single station passive ranging can be obtained only on the basis of two Doppler frequency shift measurement without using direction finding technique.

After deducing the ranging method based on the commutation relation between frequency shift and path difference, the results of this paper are compared with the existing ranging method based on frequency shift difference processing. Apart from the need for analysis itself, one reason for doing this is to fill in missing in previous studies. The original research paper only gives the ranging formula without measuring error. However, for the consideration of chapter arrangement, the content of the ranging error based on frequency shift difference processing is omitted. Another reason is that the author found in the writing process that the moving single-station Doppler frequency shift ranging has the characteristics of frequency shift exchange, mean effect and result lag. Frequency shift swap means that in the case of only two detections, in order to accurately solve the ranging value at the end position, the Doppler frequency shift value at the starting position 1 is used to replace the frequency shift value at the end position 2. Mean effect and result lag mean that the average value of two frequency shift measurements is needed to obtain a more accurate ranging value at the initial measurement location.

2. Transformation From Frequency Shift to Path Difference

2.1 Basic definitions

A Doppler receiver *R* is installed on the moving platform to detect the stationary or slow-moving target *T* on the ground, and the received Doppler frequency shift is

$$\lambda f_d = v \cos\beta \qquad (1)$$

Where: f_d is Doppler frequency shift; λ the wavelength; v the moving speed of the moving platform; β leading angle.

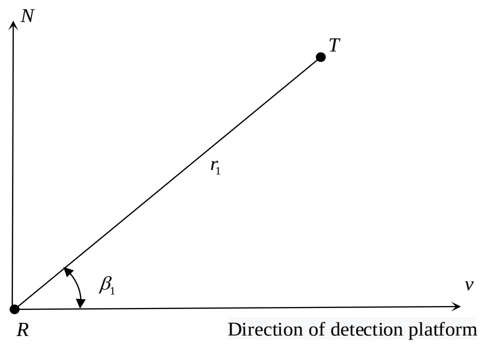

Fig. 1. Doppler frequency shift detection of moving single station.

2.2 Differential treatment of change rate of radial distance

According to the relationship between the change rate of radial distance and the radial velocity, as well as between the radial velocity and the Doppler frequency shift, the relationship between the Doppler frequency shift and the change rate of radial distance can be obtained

$$\frac{\partial r(t)}{\partial t} = v_r = v\cos\beta = \lambda f_d \qquad (2)$$

Assuming that the change of time is transient, by means of difference calculation, the differential of distance to time can be transformed into

$$\frac{\partial r(t)}{\partial t} \approx \frac{\Delta r}{\Delta t} \qquad (3)$$

Where, Δr is the path difference. Δt is the time difference taken by the platform to move from position 1 to position 2.

$$\Delta t = \frac{d}{v} \qquad (4)$$

The Doppler frequency shift expression based on the path difference measurement is obtained

$$f_d = \frac{v\Delta r}{\lambda d} \qquad (5)$$

If the Doppler shift is measured, the virtual path difference can be obtained

$$\Delta r = \frac{\lambda d}{v} f_d \qquad (6)$$

3. Ranging Solution

3.1 Detection model

Suppose there is a single station moving in a straight line, as shown in Fig. 2, from position 1 to position 2. In this process, Doppler frequency shift detection is performed twice in a row at a single moving station.

$$\lambda f_{d1} = v\cos\beta_1 \qquad (7)$$

$$\lambda f_{d2} = v\cos\beta_2 \qquad (8)$$

When the wavelength and moving speed are determined, the leading angle can be obtained according to the measured Doppler frequency shift

$$\beta_1 = \arccos\left(\frac{\lambda f_{d1}}{v}\right) \qquad (9)$$

$$\beta_2 = \arccos\left(\frac{\lambda f_{d2}}{v}\right) \qquad (10)$$

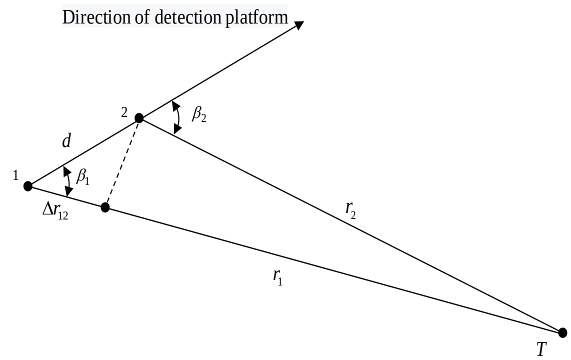

Fig. 2. Detection of fixed target by moving single station.

3.2 Ranging solution based on virtual range difference and distance ratio

From the perspective of engineering application, based on the Doppler frequency shift obtained by the second detection, the virtual path difference is given on the premise of real-time detection of the current location distance information

$$\Delta r = r_1 - r_2 = \frac{\lambda d}{v} f_{d2} \qquad (11)$$

The ratio of radial distance at two detection positions can be obtained according to sine theorem

$$\frac{r_1}{r_2} = \frac{\sin\beta_2}{\sin\beta_1} \qquad (12)$$

So, by combining the equations, the target distance r_2 at the current position can be obtained by eliminating an unknown variable r_1

$$r_2 = \frac{\lambda d f_{d2}}{v\left(\frac{\sin\beta_2}{\sin\beta_1} - 1\right)} \qquad (13)$$

The leading angle contained in the formula can be obtained by using Equation (9-10).

3.3 Simulation Calculation

During the simulation calculation, the radial distance $r_1 = 600\text{km}$ from the detection platform to the target is preset at the starting position 1. In addition, the preset detection platform moving distance $d = 100\text{km}$ and moving speed $v = 300m/s$, as well as the detection signal wavelength $\lambda = 0.3m$.

Then, the leading angle at the starting point 1 is linearly changed in the range of $0^0 < \beta_1 < 90^0$, and the radial distance r_2 of the target at the second position 2 and other geometric parameters are calculated by using trigonometric function relations. According to the definition (1) of Doppler frequency shift, the Doppler frequency shift value is calculated and obtained.

On this basis, the calculated value of radial distance r_2 , Eq. (13), obtained based on frequency shift detection is compared with the theoretical value of radial distance obtained by using trigonometric function. The simulation results show that, if the Doppler frequency shift value at the detection position 1 or 2 is directly used to calculate, the ranging solution has a large calculation error. Better results can be obtained if the average Doppler shift at two positions is used. Therefore, the ranging equation (13) is modified as

$$r_2 = \frac{\lambda d\left(f_{d1} + f_{d2}\right)}{2v\left(\frac{\sin\beta_2}{\sin\beta_1} - 1\right)} \qquad (14)$$

The ranging equation (14) is further transformed into a function based only on Doppler frequency shift

$$r_2 = \frac{\lambda d\left(f_{d1} + f_{d2}\right)}{2v\left(\frac{\sin\beta_2}{\sin\beta_1} - 1\right)} = \frac{\lambda d\left(f_{d1} + f_{d2}\right)}{2v\left(\frac{\sqrt{1-\cos^2\beta_2}}{\sqrt{1-\cos^2\beta_1}} - 1\right)} = \frac{\lambda d\left(f_{d1} + f_{d2}\right)}{2v\left(\frac{\sqrt{1-\left(\frac{\lambda}{v} f_{d2}\right)^2}}{\sqrt{1-\left(\frac{\lambda}{v} f_{d1}\right)^2}} - 1\right)} \qquad (15)$$

Fig. 3 shows the relative calculation errors of ranging solutions at different moving lengths, from which it can be seen that shorter moving distances can achieve better calculation accuracy and within a wide range, the relative calculation errors can be kept less than 3%.

Fig. 4 shows the relative calculation error of ranging solutions at different detection distances. It can be seen that the relative calculation error is inversely proportional to the radial distance and can be kept less than 3% within a wide range.

The simulation results also show that the values of flight speed and wavelength have no influence on the analysis of relative calculation error.

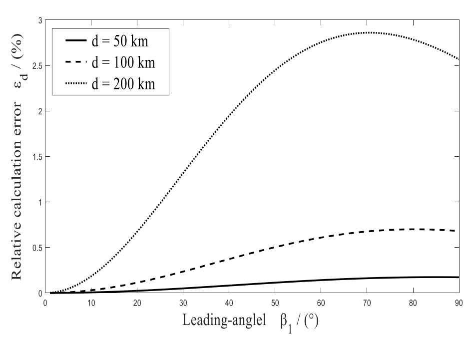

Fig. 3. Relative calculation error at different moving distances.

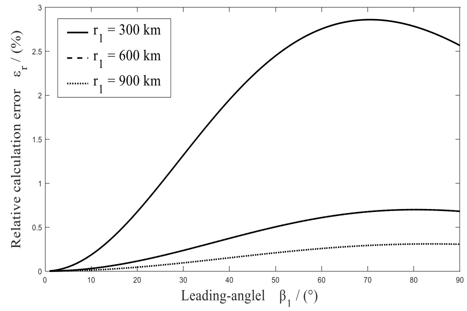

Fig. 4. The relative calculation error of different target distance.

4. Ranging Error

4.1 Differential analysis

Previous studies have shown that the RMS of travel distance, wavelength and flight speed have little effect on the relative ranging error. Therefore, this paper only analyzes the ranging error generated by Doppler frequency shift measurement. The relative ranging error is analyzed by total differential method. First set

$$r_2 = \frac{a(f_{d1} + f_{d2})}{W} \quad (16)$$

Thereinto

Constant: $a = \frac{\lambda d}{2v}$

The transition function:

$$W = \left(\frac{P}{Q} - 1\right)$$

$$P = \sqrt{1 - \left(\frac{\lambda}{v} f_{d2}\right)^2}$$

$$Q = \sqrt{1 - \left(\frac{\lambda}{v} f_{d1}\right)^2}$$

Range error resulting from frequency shift f_{d1}

$$\frac{\partial r_2}{\partial f_{d1}} = \frac{a}{W} - \frac{a(f_{d1} + f_{d2})}{W^2} \frac{\partial W}{\partial f_{d1}} \quad (17)$$

Thereinto

$$\frac{\partial W}{\partial f_{d1}} = -\frac{P}{Q^2} \frac{\partial Q}{\partial f_{d1}}$$

$$\frac{\partial Q}{\partial f_{d1}} = -\left(\frac{\lambda}{v}\right)^2 \frac{f_{d1}}{\sqrt{1 - \left(\frac{\lambda}{v} f_{d1}\right)^2}}$$

Range error resulting from frequency shift f_{d2}

$$\frac{\partial r_2}{\partial f_{d2}} = \frac{a}{W} - \frac{a(f_{d1}+f_{d2})}{W^2}\frac{\partial W}{\partial f_{d2}} \quad (18)$$

Thereinto

$$\frac{\partial W}{\partial f_{d2}} = \frac{1}{Q}\frac{\partial P}{\partial f_{d2}}$$

$$\frac{\partial P}{\partial f_{d2}} = -\left(\frac{\lambda}{v}\right)^2 \frac{f_{d2}}{\sqrt{1-\left(\frac{\lambda}{v}f_{d2}\right)^2}}$$

4.2 Error curve

When the error of each observation is zero mean, independent of each other, relative ranging error

$$\sigma_r = \frac{\sigma_f}{r_2}\left[\left|\frac{\partial r_2}{\partial f_{d1}}\right| + \left|\frac{\partial r_2}{\partial f_{d2}}\right|\right] \quad (19)$$

Where, σ_f is the root mean square error of frequency shift measurement error.

The geometric parameters and Doppler shift are set and calculated in the same way as in the simulation. Parameter value used in calculation

$r_1 = 600\text{km}$, $d = 100\text{km}$, $v = 300m/s$, $\lambda = 0.3m$, $\sigma_f = 50Hz$ 。

Fig. 5 shows the relative ranging error curves with different RMS of Doppler shift measurement errors. Fig. 6 shows the relative ranging error curves at different moving distances. Fig. 7 shows the relative ranging error curves at different radial distances. The calculation results show that the relative ranging error of less than 5% can be achieved by using the reciprocal relation between frequency shift and path difference when the leading angle is more than 30 degrees. Obviously, the airborne positioning method based on frequency measurement will support side-view detection.

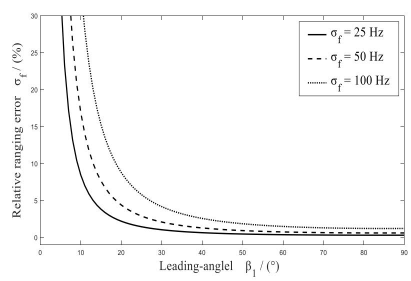

Fig. 5. The relative ranging error of different frequency shift measurement error.

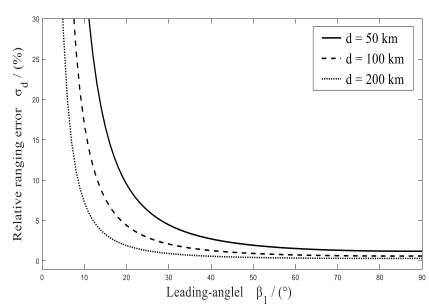

Fig. 6. Relative ranging error at different moving distances.

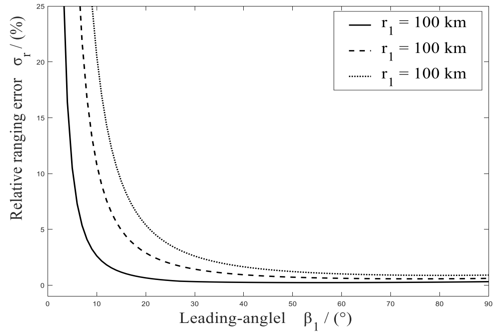

Fig. 7. Relative ranging errors at different radial distances.

5. Comparison With the Ranging Solution Based on Frequency Shift Differential Processing

5.1 Existing studies

This section first retells the mathematical derivation results of previous studies [4]-[5]. Still based on the geometric model shown in Fig. 2, it is assumed that the Doppler change rate detected by the detection plat form at position 1 after moving the distance d uniformly along a straight line is

$$\dot{f}_d = \frac{v_{t1}^2}{\lambda \cdot r_1} \qquad (20)$$

Where, v_{t1} is the tangential velocity of the detection platform; r_1 the radial distance between the target and the detection platform.

From the mathematical definition, the rate of Doppler change can be approximated by the measured value of Doppler frequency difference between two detection endpoints during a time period Δt

$$\dot{f}_d = \frac{\Delta f_d}{\Delta t} = \frac{f_{d2} - f_{d1}}{\Delta t} \qquad (21)$$

By integrating equations (20) and (21), and using the relationship $v^2 = v_r^2 + v_t^2$ between velocity vector and its components, and the relationship $v_r = \lambda f_d$ between radial velocity and Doppler frequency shift, the ranging formula as follows can be obtained

$$r_1 = \frac{\left(v^2 - \lambda^2 f_{d1}^2\right)\Delta t}{\lambda\left|\Delta f_d\right|} = \frac{d\left(v^2 - \lambda^2 f_{d1}^2\right)}{v\lambda\left|\Delta f_d\right|} \qquad (22)$$

Where: $\Delta f_d = f_{d1} - f_{d2}$.

5.2 Simulation Modification

Parameter value used in calculation

$r_1 = 600\text{km}$, $d = 100\text{km}$, $v = 300m/s$, $\lambda = 0.3m$, $\sigma_f = 50Hz$.

The radial distance and Doppler frequency shift values at the two detection positions are respectively selected to calculate. Fig. 8 shows some of the calculated curves selectively.

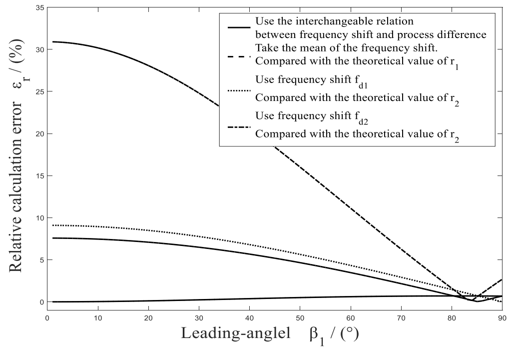

Fig. 8. Comparison of relative calculation errors of two ranging methods.

The results show that, assuming that the detection platform moves the distance d uniformly along a straight line, if the average frequency shift at two positions is adopted, the ranging formula at position 1 has the minimum relative calculation error, that is,

$$r_1 = \frac{d\left(v^2 - 0.25\lambda^2\left(f_{d1} + f_{d2}\right)^2\right)}{v\lambda\left|\Delta f_d\right|} \quad (23)$$

The second curve in the legend of Fig. 8 shows the relative calculation error of Equation (23). This shows mathematically that there is a lag in the detection of target distance. It is necessary to obtain the average value of Doppler frequency shift on a moving distance, and then only the radial distance at the starting position can be accurately obtained.

The fourth curve in the legend of Fig. 8 shows the relative calculation error of equation (24), which indicates that the Doppler frequency shift at the terminating position is used to calculate the radial distance at position 2, and the relative calculation error obtained is very poor

$$r_2 = \frac{d\left(v^2 - \lambda^2 f_{d2}^2\right)}{v\lambda\left|\Delta f_d\right|} \quad (24)$$

If equation (24) is simply modified

$$r_2 = \frac{d\left(v^2 - \lambda^2 f_{d1}^2\right)}{v\lambda\left|\Delta f_d\right|} \quad (25)$$

The third curve in the legend of Fig. 8 shows the relative calculation error of Equation (25). The modified results show that, in the case of only two detections, in order to directly use the Doppler shift measurement value at the current moving distance to solve the radial distance at the end position more accurately, the Doppler shift value at the starting position 1 should be used to replace the frequency shift value at the second position.

The first curve in the legend of Fig. 8 shows the relative calculation error of the ranging solution based on frequency shift and path difference transformation at the same moving distance. Obviously, the relative calculation error of the ranging solution based on frequency shift and path difference transformation is far less than that based on frequency shift differential processing.

5.3 Comparison of relative ranging errors

Preset preestablish

$$r_2 = a_0 \frac{P_0}{Q_0} \qquad (26)$$

Thereinto

$a_0 = \frac{d}{v\lambda}$

$P_0 = v^2 - \lambda^2 f_{d1}^2$

$Q_0 = |\Delta f_d|$

Range error resulting from frequency shift f_{d1}

$$\frac{\partial r_2}{\partial f_{d1}} = \frac{a_0}{Q_0^2}\left(Q_0 \frac{\partial P_0}{\partial f_{d1}} - P_0 \frac{\partial Q_0}{\partial f_{d1}}\right) \qquad (27)$$

Thereinto

$\frac{\partial P_0}{\partial f_{d1}} = -2\lambda^2 f_{d1}$

$\frac{\partial Q_0}{\partial f_{d1}} = 1$

Range error resulting from frequency shift f_{d2}

$$\frac{\partial r_2}{\partial f_{d2}} = -\frac{a_0 P_0}{Q_0^2} \frac{\partial Q_0}{\partial f_{d2}} \qquad (28)$$

Thereinto

$\frac{\partial Q_0}{\partial f_{d2}} = 1$

Based on formula (19) for calculating the relative ranging error, Fig. 9 shows the comparison of the relative ranging error of the two methods. Generally speaking, the formula of the two methods is basically the same.

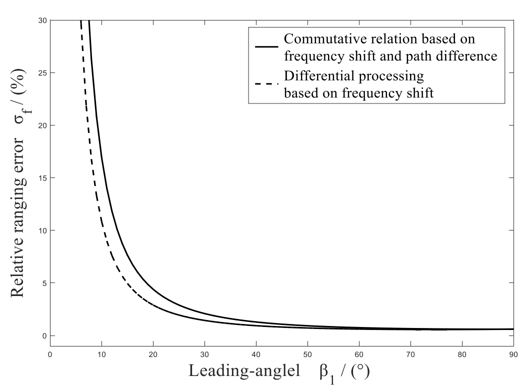

Fig. 9. Comparison of two ranging errors.

6. Conclusion

Compared with the existing ranging solutions based on frequency shift differential processing, the ranging solutions presented in this paper can obtain better computing accuracy. However, the analysis results show that although the ranging method based on frequency shift and path difference exchange relation is different from the ranging method based on frequency shift difference treatment. One is the differential treatment of radial distance based on Doppler frequency shift equation. The other is the differential processing of frequency shift based on Doppler rate equation. However, the relative ranging errors obtained by the two methods are basically the same. Does this indicate that different data processing methods cannot effectively change the error performance of ranging in the case of the same basic physical measurement methods.

The practice of using simulation to modify the result of mathematical derivation shows that the analysis of Doppler frequency shift ranging may not be complete. The more significant result of this article is that in the process of retelling the existing ranging solution based on frequency shift differential processing, through simulation calculation, it is revealed that the Doppler frequency shift ranging of a moving single station has the characteristics of frequency shift interchange, mean effect and result lag, although the process described may not be complete, it is still simple.

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Tao Yu has been engaged in the research of passive positioning technology for more than ten years and has published more than 200 papers and a Chinese Science and Technology Monograph. More than ten invention patents have been applied for and granted.

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